Physics 201 60 Name KEY

Exam 2 – Capacitance, Circuits and Magnetism

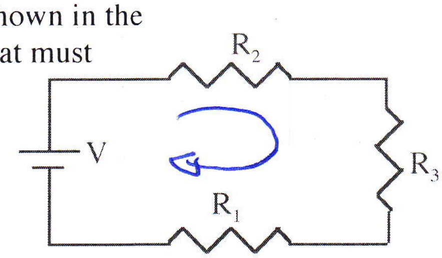
March 11, 2010

This is a closed book examination but during the exam you may refer to a 4"x6" note card with words of wisdom you have written on it. There is extra scratch paper available. Please explain your answers. Your explanation is worth 3/4 of the points on all questions.

A general reminder about problem solving:

- Focus
  - Draw a picture of the problem
  - What is the question? What do you want to know?
  - List known and unknown quantities
  - List assumptions
- Physics
  - Determine approach – What physics principles will you use?
  - Pick a coordinate system
  - Simplify picture to a schematic (if needed)
- Plan
  - Divide problem into sub-problems
  - Modify schematic and coordinate system (if needed)
  - Write general equations
- Execute
  - Write equations with variables
  - Do you have sufficient equations to determine your unknowns?
  - Simplify and solve
- Evaluate
  - Check units
  - Why is answer reasonable?
  - Check limiting cases!
- Show All Your Work!

1) [4 PTS] You connect three light bulbs (resistors) to a battery as shown in the diagram to the right. If the light bulb labled R_2 is brightest, what must be true?

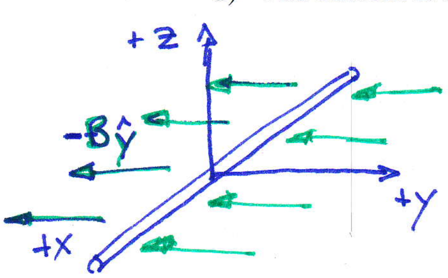


a) R_2 has the largest resistance.
b) R_2 is first resistor so it has largest current through it.
c) R_2 has the smallest resistance.
d) R_2 has the smallest voltage drop.

The current is the same through all objects that are in series.
Brightness = Power $= VI = I^2R$ (since $V \approx IR$) so largest R is brightest

2) [4 PTS] A current carrying wire is placed between the poles of a strong magnet. The magnetic field is in the $-y$ direction and the wire is along the x-axis. If the wire is deflected in the $+z$ direction what is the direction of the current?

a) The current is traveling in the $-x$ direction.
b) The current is traveling in the $+x$ direction.
c) The current is not moving but there is a net negative charge on the wire.
d) The current is not moving but there is a net positive charge on the wire.



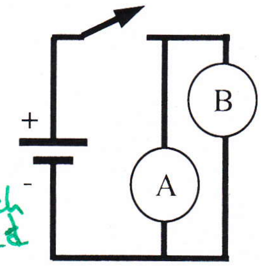
$\vec{F} = q\vec{v} \times \vec{B} = \ell \vec{I} \times \vec{B}$

$F_z = \ell\, {}^{-}I_x \times {}^{-}B_y$

Right Rule
$\hat{x} \times \hat{y} = \hat{z}$

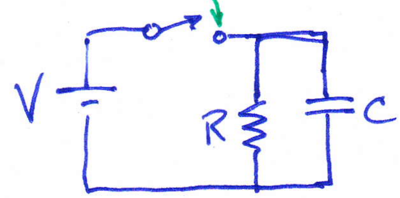
Current must be in $-\hat{x}$ direction

The next two problems involve the circuit to the right. A capacitor is placed at position B and a light bulb is placed at position A. Assume the voltage source can provide any current.



3) [4 PTS] The capacitor is initially uncharged. When the switch is closed what happens?
   a) The light bulb does not light.
   b) The light bulb starts off dim and then gets brighter.
   c) The light bulb turns on and is a constant brightness.
   d) The light bulb starts off bright and then gets dimmer.

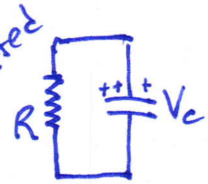
switch closed



If Voltage source can supply the needed current then I is constant through resistor $I = \frac{V}{R}$ NOTE: The current diminishes though the capacitor as it charges (but doesn't effect light bulb)

4) [4 PTS] After a long time the switch is then opened. What happens?
   a) Nothing. The light bulb was off and it stays off.
   b) The light bulb starts off dim and then gets brighter.
   c) Nothing. The light bulb turns on and stays on.
   d) The light bulb starts off bright and then gets dimmer.

Battery Disconnected



Capacitor charge (voltage) ~~past~~ creates current through resistor

$V_R = V_c = V_0 e^{-t/\tau}$ $\tau = RC$

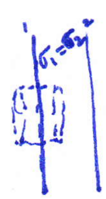
5) [4 PTS] You are measuring the voltage across a capacitor with a charge Q on it. How does the voltage change when you insert a dielectric with κ=2 into the capacitor.
   a) The voltage decreases.
   b) The voltage does not change.
   c) The voltage increases.

$Q = C_0 V$ $C = \frac{K\epsilon_0 A}{d}$ For parallel plate capacitor

The charge is constant so if Capacitance changes the voltage will change.

$K=1$ for air so $C = 2C_0$ since C increase V decreases ($C_0 = \frac{\epsilon_0 A}{d}$)

NOTE: for a given breakdown voltage you can store more charge



$E_1 2A = \frac{\sigma A}{\epsilon}$ $E_1 = \frac{\sigma}{2\epsilon}$

for two plates add fields $E = \frac{\sigma}{\epsilon}$

$V = \int E \cdot dr$ $\frac{V}{d} = \frac{Q}{A\epsilon}$

$C = \frac{Q}{V} = \frac{A\epsilon}{d}$

6) [4 PTS] A parallel plate capacitor is charged and stores a total energy of U_i. You decide to increase the plate separation by 4 (i.e. they were separated by 1 mm and now they are separated by 4 mm). What is the new energy stored in the capacitor?
   a) $U_f = ½U_i$
   b) $U_f = ¼U_i$
   c) $U_f = U_i$
   d) $U_f = 2U_i$
   e) $U_f = 4U_i$
   f) $U_f = 16U_i$

$U = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C}$

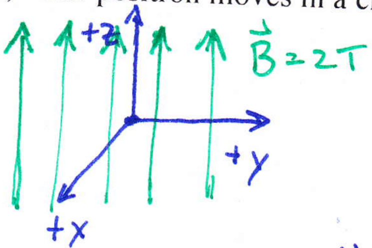
$C = \frac{1}{4}C_0$ $U_0 = \frac{1}{2}\frac{Q_0^2}{C_0}$ $U_f = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{Q_0^2}{\left(\frac{C_0}{4}\right)} = 4\left(\frac{1}{2}\frac{Q_0^2}{C_0}\right)$

$Q = Q_0$ $U_f = 4U_0$

It took energy to pull plates further apart

7) [4 PTS] The radioactive decay of ^{18}F produces a positron that is initially at <u>rest</u> in a vacuum chamber. The positron (a positively charged electron) is in the middle of a uniform magnetic field of magnitude 2 Teslas. The field is in the $+z$ direction where the gravitational acceleration is in the $-z$ direction. How does the positron move?

a) The positron moves in a circle with an angular velocity vector in the $+z$ direction.
b) The positron moves in the $-z$ direction.
c) The positron moves in the $+z$ direction.
d) The positron moves in a circle with an angular velocity vector in the $-z$ direction.



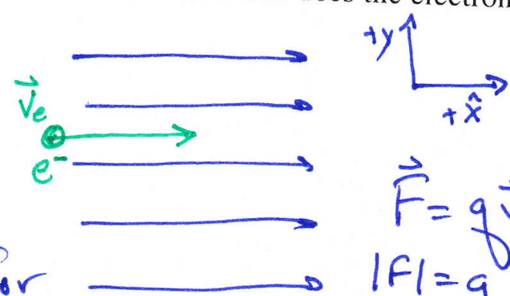
$\vec{F} = q\vec{v} \times \vec{B}$ $\quad \vec{F} = mg$

$|V| = 0$ so the positron has no force due to $\vec{B}$ but will move due to gravity

NOTE: $\vec{V}_z \times \vec{B} = 0$ so F_m will stay $\emptyset$

8) [4 PTS] An electron moving in the $+x$ direction enters a region of uniform magnetic field that is also oriented in the $+x$ direction. In which direction does the electron feel a force?

a) The $+y$ direction.
b) The $+z$ direction.
c) The $-y$ direction.
d) The $-z$ direction.
e) None of the above.



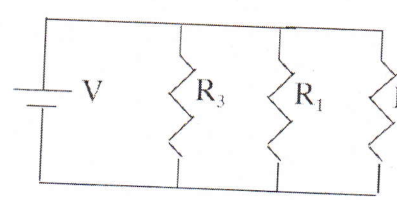
$\vec{F} = q\vec{v} \times \vec{B} = \cancel{q}$

$|F| = q|V||B|\sin\theta$

Cross product for two parallel vectors is <u>zero</u>.

9) [4 PTS] For the circuit to the right the light bulb labled R_3 is the brightest and the light bulb labled R_1 is the dimmest. The circuit is now changed to resemble the circuit in question #1. How does the brightness of the light bulbs change?

a) R_2 is now the brightest light bulb.
b) R_3 is still the brightest light bulb.
c) R_1 is now the brightest light bulb.
d) All the light bulbs are of equal brightness.



Power = Brightness

$P = VI = \frac{V^2}{R}$ since $V = IR$ Brightest will have smallest Resistance

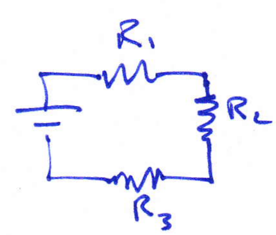
* Voltages are all the same since they are in parallel

In parallel

$P_3 > P_2 > P_1$

$R_3 < R_2 < R_1$

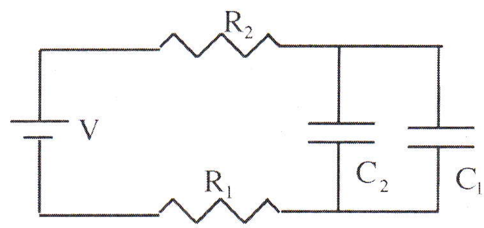
$\therefore$ Now switch so resistors are in series current is the same so



$P = I^2R \quad P_3 < P_2 < P_1$

Brightest was dimmest!

10) [24 PTS] You construct the circuit shown to the right with R_1= 250Ω, R_2=750Ω, C_1=40μF, C_2=10μF and V=3Volts. The capacitors are initially uncharged. NOTE: When graphing V(t) and I(t) make sure to label both axis with actual values.



a) What is the time constant for the entire circuit?
b) Which resistor has the largest voltage drop across it?
c) Which resistor has the largest current through it?
d) Graph the voltage across C_2 as a function of time.
e) Graph the current through C_1 as a function of time.
f) Graph the current through R_1 as a function of time.
g) How long does it take C_1 to charge to 2 volts?
h) What is the charge on each capacitor at t=75 msec?

$Q_1 = 93.2\,\mu C$
$Q_2 = 23.3\,\mu C$

(a) $\tau = RC$

$R_{TOT} = R_1 + R_2 = 250\,\Omega + 750\,\Omega = 1\,k\Omega$

$C_{TOT} = C_1 + C_2 = 40\,\mu F + 10\,\mu F = 50\,\mu F$

$RC = 1 \times 10^3\,\Omega \cdot 50 \times 10^{-6}\,F = 50 \times 10^{-3}\,s$

$\tau = 50$ msec

(b) In series I is constant (same)

$V = IR$ so $R_2 > R_1$ and $V_2 > V_1$

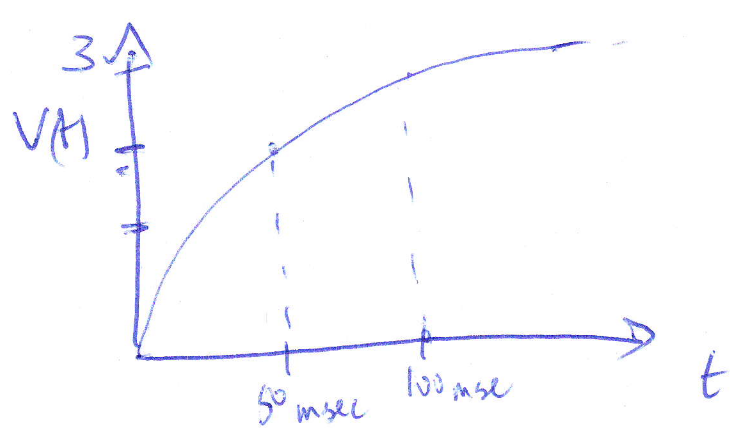
R_2 has greatest voltage drop

(c) I is the same through both resistors since they are in series

(d) $V_{C2} = V_{C1}$ since they are in parallel

$V(t) = V_0\left(1 - e^{-t/\tau}\right)$ $V_0 = 3$ Volts

$\tau = 50$ msec

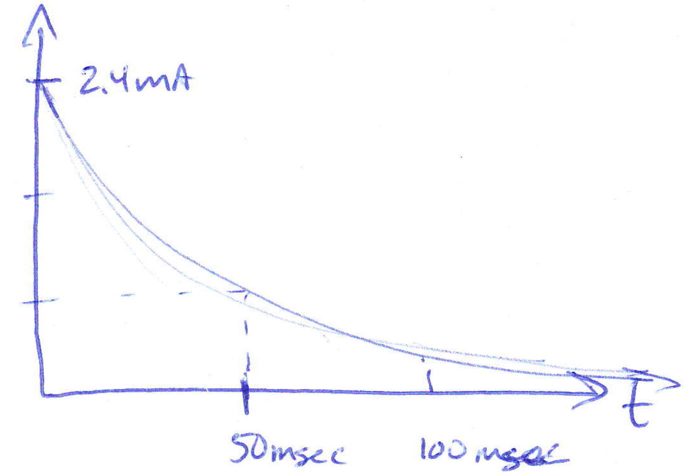


(e) $Q = CV$ so $I = \frac{dQ}{dt} = C\frac{dV}{dt}$

$I_{C_1}(t) = C_1 \cdot \frac{d}{dt} V(t) = -C_1 V_0\left(-\frac{1}{\tau}\right) e^{-t/\tau}$

$= \frac{V_0}{R_{TOT}} \cdot \frac{C_1}{(C_1+C_2)} e^{-t/\tau} = \frac{3V}{1k\Omega} \cdot \frac{4}{5} e^{-t/\tau}$

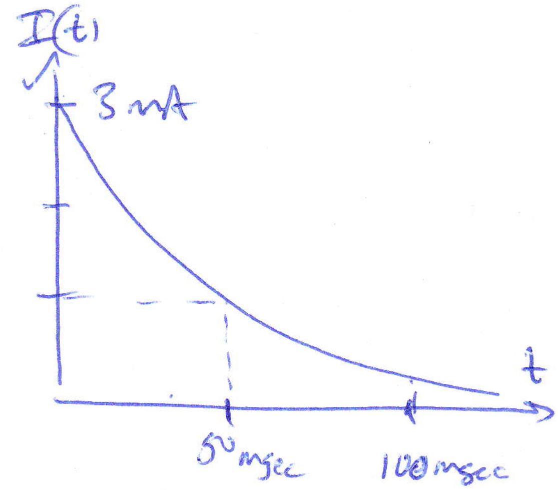
$I_{C_1}(t) = 2.4\,mA\, e^{-t/\tau}$



(f) $V = IR$ so $I_{initial} = \frac{\Delta V_{TOT}}{R_{TOT}}$

$\Delta V_{TOT}(t) = V_0 - V(t) = V_0 e^{-t/\tau}$

$I(t) = \frac{V_0}{R_{TOT}} e^{-t/\tau} = 3mA\, e^{-t/\tau}$



(g) $2\,\text{Volts} = 3\,\text{Volts}\left(1 - e^{-t/\tau}\right)$

$3Ve^{-t/\tau} = 1V$ so $e^{-t/\tau} = \frac{1}{3}$ and $\ln(e^{t/\tau}) = \ln\frac{1}{3}$

$\frac{t}{\tau} = \ln 3$ so $t = 50\,msec \cdot (\ln 3) = 54.9\,msec$

(h) $Q = CV$

$Q_1 = C_1\, 3V\left(1 - e^{-75msec/50msec}\right)$ $\quad Q_2 = C_2\, 3V\left(1 - e^{-75msec/50msec}\right)$